Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – calculus

Wednesday 18 May 2016 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[60 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

The function f is defined by $f(x) = \mathrm{e}^x \sin x$, $x \in \mathbb{R}$.

(a) By finding a suitable number of derivatives of f, determine the Maclaurin series for $f(x)$ as far as the term in x^3. [7]

(b) Hence, or otherwise, determine the exact value of $\lim\limits_{x \to 0} \dfrac{\mathrm{e}^x \sin x - x - x^2}{x^3}$. [3]

(c) The Maclaurin series is to be used to find an approximate value for $f(0.5)$.

(i) Use the Lagrange form of the error term to find an upper bound for the absolute value of the error in this approximation.

(ii) Deduce from the Lagrange error term whether the approximation will be greater than or less than the actual value of $f(0.5)$. [7]

2. [Maximum mark: 7]

A function f is given by $f(x) = \int_0^x \ln(2 + \sin t)\,\mathrm{d}t$.

(a) Write down $f'(x)$. [1]

(b) By differentiating $f(x^2)$, obtain an expression for the derivative of $\int_0^{x^2} \ln(2 + \sin t)\,\mathrm{d}t$ with respect to x. [3]

(c) Hence obtain an expression for the derivative of $\int_x^{x^2} \ln(2 + \sin t)\,\mathrm{d}t$ with respect to x. [3]

3. [Maximum mark: 9]

(a) Given that $f(x) = \ln x$, use the mean value theorem to show that, for $0 < a < b$,

$$\frac{b-a}{b} < \ln\frac{b}{a} < \frac{b-a}{a}.$$ [7]

(b) Hence show that $\ln(1.2)$ lies between $\frac{1}{m}$ and $\frac{1}{n}$, where m, n are consecutive positive integers to be determined. [2]

4. [Maximum mark: 13]

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{y} - xy$ where $y > 0$ and $y = 2$ when $x = 0$.

(a) Show that putting $z = y^2$ transforms the differential equation into $\frac{\mathrm{d}z}{\mathrm{d}x} + 2xz = 2x$. [4]

(b) By solving this differential equation in z, obtain an expression for y in terms of x. [9]

5. [Maximum mark: 14]

Consider the infinite series $S = \sum_{n=0}^{\infty} u_n$ where $u_n = \int_{n\pi}^{(n+1)\pi} \frac{\sin t}{t}\,\mathrm{d}t$.

(a) Explain why the series is alternating. [1]

(b) (i) Use the substitution $T = t - \pi$ in the expression for u_{n+1} to show that $|u_{n+1}| < |u_n|$.

(ii) Show that the series is convergent. [9]

(c) Show that $S < 1.65$. [4]